

HILLSBOROUGH TOWNSHIP SCHOOL DISTRICT

MATHEMATICS CURRICULUM

AP Calculus AB

July, 2020

Course Overview

AP Calculus AB

AP Calculus AB is offered to students who have been successful in Math Analysis Honors. During this course students explore, visualize, analyze, and calculate according to the topics referenced by the *AP Calculus Course Description*, and are also exposed to several supplementary topics such as L'Hopital's Rule that are not required parts of the course. Concepts are investigated analytically, graphically, and numerically. Students are expected to relate various representations to each other. The teachers of this course have also developed additional long-term assignments to aid students in preparation for taking the AP Exam. Students gain a thorough understanding of both Differential Calculus and Integral Calculus through various exercises provided by the course textbook as well as accompanying classroom investigations and activities as outlined below.

Throughout the course students will use a variety of strategies and tools to gain a solid understanding of the material. Technology which includes computer work, graphing calculators and exploring web based sites will enhance the educational experience by allowing lessons to become more interactive and inquiry-based.

The course is structured around the New Jersey Student Learning Standards. The curriculum will reflect various teaching strategies and offer opportunities for enrichment and reinforcement based on individual need.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 1 Foundations & Limits	Timeframe/Pacing: 18 days
<p>Essential Questions</p> <ul style="list-style-type: none"> ● How can you describe the behavior of a function as x approaches a certain value? ● How can you determine the continuity of a function at a point? ● How can you determine the end behavior of a function? 	
<p>Enduring Understandings</p> <ul style="list-style-type: none"> ● The limit of a function indicates the value that the function tends toward as x approaches a certain value. ● Limits can be found graphically, numerically, and analytically. ● Reasoning with definitions, theorems, and properties can be used to justify claims about limits. 	
<p>Standards Taught and Assessed</p> <ul style="list-style-type: none"> ● F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. 	
<p>Highlighted Interdisciplinary Connections</p> <ul style="list-style-type: none"> ● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. ● ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest. 	
<p>Highlighted Career Ready Practices and 21st Century Themes and Skill</p> <ul style="list-style-type: none"> ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions 	
<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness). 	
<p>Pre-Assessment</p> <ul style="list-style-type: none"> ● Determine the symmetry of a graph or function. ● Express the domain of a function using interval notation. 	<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<ul style="list-style-type: none"> ● Use properties of trigonometry to evaluate an expression and solve an equation. ● Simplify a difference quotient algebraically. ● Write a mathematical function that models a real-world situation. 				
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Apply properties of various classes of mathematical functions and skills involved in function/expression manipulations.	SMP 5 Use appropriate tools strategically.	Evaluate various trigonometric functions. Draw a graph of a given piecewise function.	Label Trigonometric unit circle with coordinates and angles.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Understand the tangent line problem and area problem.	SMP 2 Reason abstractly and quantitatively.	Use limits to approximate the slope of $f(x) = x^2$ at $x = 3$.	Exploration: 1) Estimate the slope of the tangent line to the graph of $f(x) = x^2$ at (1, 1) (by finding the slope between (1,1) and various points. 2) Approximate the area of the region bounded by $f(x)=x^2$, $y=0$, $x=1$ by using 5 inscribed rectangles and 5 circumscribed rectangles.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Evaluate a limit using a numerical or graphical approach.</p>	<p>SMP2 Reason abstractly and quantitatively.</p>	<p>Create a table of values to approximate the value of</p> $\lim_{x \rightarrow 2} \frac{4\sqrt{x} + 1}{x^{\frac{3}{2}}}$	<p>Create Representations: Present students with a limit expression in analytical form), and then have them translate that expression into a variety of representations: constructing a graph, creating a table of values, and writing it as a verbal expression. Then have students check their graphs and tables using technology.</p>	
<p>Learn different ways that a limit can fail to exist.</p>	<p>SMP3 Construct viable arguments and critique the reasoning of others.</p>	<p>Give an example of a limit that does not exist.</p>	<p>Class discussion and paired practice solving similar examples.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan</p>
<p>Evaluate a limit analytically using a variety of strategies (applying properties of limits, factoring, rationalizing technique).</p>	<p>SMP7 Look for and make use of structure.</p>	<p>$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$</p>	<p>Work Backward: Present students with a set of limit problems. Rather than determining the given limits, have them make a list of the various strategies that would be used to determine the limits (e.g., factoring, multiplying by conjugate, and simplify using trigonometric identities). After confirming their list is complete, have students work in pairs to create and write limit</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			problems, each requiring one of the listed strategies. Then have them swap problems with another pair of students to complete each other's problems.	
Determine limits using the Squeeze Theorem.	SMP7 Look for and make use of structure.	$\lim_{x \rightarrow 0} x \cos x$	Graphing calculator activity and discussion.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan
Apply properties of continuity (to determine continuity at a point and determine continuity on closed and open intervals) and evaluate one sided limits.	SMP3 Construct viable arguments and critique the reasoning of others.	Prove whether or not the function $f(x) = \frac{x}{x^2 - x}$ is continuous at $x = 0$.	Discussion Groups: Give each group of students a piecewise-defined function, a graph paper, and a list of x-values. Have them graph the function, then discuss whether the function is continuous or discontinuous at each x-value, and explain why. Ask students to take turns recording the group's conclusion for each x-value. If continuous, have students discuss and show that all three continuity conditions are satisfied. If discontinuous, have students state which condition was not satisfied.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Apply the Intermediate Value Theorem.</p>	<p>SMP2 Reason abstractly and quantitatively.</p>	<p>Find a value for b for which the Intermediate Value Theorem applies for the function $f(x) = x^2 - 3x + 2$ on the interval $[-1, b]$. State a conclusion that can be made using this value of b.</p>	<p>Think Aloud: In small groups, have students discuss the Intermediate Value Theorem and share ideas about real-world applications (e.g., speed of your car and weight of your kitten). Have groups post their ideas on a classroom wall using sticky notes.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan</p>
<p>Determine infinite limits and find vertical asymptotes.</p>	<p>SMP5 Use appropriate tools strategically.</p>	<p>Find any vertical asymptotes of $f(x) = \frac{x}{x^2 - x}$</p>	<p>Warm-up activity to how to find vertical asymptotes; class discussion.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan</p>
<p>Evaluate limits at infinity and find horizontal asymptotes.</p>	<p>SMP5 Use appropriate tools strategically.</p>	<p>Find any horizontal asymptotes of $f(x) = \frac{x}{x^2 - x}$ $f(x) = \frac{x}{x^2 - x}$</p>	<p>Warm-up activity to how to find horizontal asymptotes; class discussion.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Benchmark Assessment <ul style="list-style-type: none">• Not Applicable	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none">• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Summative Assessment(s) <ul style="list-style-type: none">• Foundations Common Assessment• Limits Common Assessment Performance Assessment: Draw a graph that incorporates numerous different given limit properties and asymptotes.	

**Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum**

Unit Title: 2 Differentiation	Timeframe/Pacing: 26 Days
<p>Essential Questions</p> <ul style="list-style-type: none"> ● How can we find the rate of change of a quantity? ● How does the rate of change of one quantity affect the rate of change of another? ● How can we visualize the rate of change of a quantity and what does that tell us? 	
<p>Enduring Understandings</p> <ul style="list-style-type: none"> ● Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. ● Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both. 	
<p>Standards Taught and Assessed</p> <ul style="list-style-type: none"> ● F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. 	
<p>Highlighted Interdisciplinary Connections</p> <ul style="list-style-type: none"> ● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. ● ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest. ● Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. 	
<p>Highlighted Career Ready Practices and 21st Century Themes and Skill</p> <ul style="list-style-type: none"> ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions 	
<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness). 	
<p>Pre-Assessment</p> <ul style="list-style-type: none"> ● Use a table of values to estimate the rate of 	<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

change of a function at a particular time and explain how limits can give a more accurate solution to the question.		<ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Apply the limit definition to find the derivative of a function.	SMP 4 Model with mathematics.	Use the limit definition of the derivative to find $f'(x)$ for $f(x) = \frac{1}{x^2}$.	Graphical exploration (using Desmos) to see transition from secant line to tangent line. Class discussion to elicit definition of a derivative; scaled examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Understand the relationship between differentiability and continuity.	SMP 2 Reason abstractly and quantitatively.	Sketch a function which is: continuous, but not differentiable at $x = a$.	Match Mine Create cards containing graph images of functions with various continuous, discontinuous, differentiable, and nondifferentiable points or intervals. Provide each student in a pair with the same nine cards. Student A arranges their graphs in a 3 x 3 grid, which is not visible to Student B. Student A describes each of their graph's positions using information about continuity and differentiability to	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			describe the graph. Based on the descriptions, Student B attempts to arrange their cards to match the grid of Student A.	
Apply basic differentiation rules (constant rule, power rule, constant multiple rule, sum and difference rule, sine and cosine).	SMP 5 Use appropriate tools strategically.	Evaluate: $\frac{d}{dx} \left[x^2 + 2x^{\frac{5}{2}} + 3 \sin x \right]$	Error Analysis: Assign a function to each student. Ask them to find the function's derivative using one or more derivative rules. Allow them to check their answers. Ask half of the class to redo their work to include an error, thus having the wrong answer. Ask students to record their correct or incorrect work on a card. Mix up the cards and redistribute, having students determine if the answer is correct or incorrect. If incorrect, they should explain what error was made, and find the correct answer.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use derivatives to find rates of change.	SMP1 Make sense of problems and persevere in solving them.	A projectile is shot upward from the surface of the Earth with an initial velocity of 120 m/s. What is the velocity after 5 seconds?	Graph and Switch: Present students with two or three functions and the graph of each function. Have each student choose a random derivative question and one function. Questions could include: Find the average rate of change	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			<p>on an interval, instantaneous rate of change at a point, derivative as a function, derivative value at a point, or equations for tangent or normal lines at a point. Have students answer their question and place their answer onto the function's graph. Then have students share their solutions with each other to give and receive feedback.</p>	
<p>Find derivatives using the Product and Quotient rules.</p>	<p>SMP 5 Use appropriate tools strategically.</p>	<p>Differentiate: $f(x) = x^2 \cos x$ $f(x) = \frac{2x}{x^3 + 1}$ $f(x) = 3 \tan x$</p>	<p>Round Table: Provide each student with the same worksheet containing four functions that require the product rule or quotient rule when finding the derivative. Then have students sit in groups of four. Each student determines the derivative of function No. 1, and then they pass their papers clockwise to the next student. Each student checks the first problem and, if necessary, discusses any mistakes with the previous student. Each student now completes function No. 2 on the paper, and the process continues until each student has their original paper back.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Evaluate higher-order derivatives.	SMP 5 Use appropriate tools strategically.	$\frac{d^2y}{dx^2}$ Find $\frac{d^2y}{dx^2}$ for $y = 3x^2 \sin x$	Teacher-led discussion using various classes of functions. Students will generalize patterns where they exist.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the derivative of a function using the Chain Rule.	MP 7 Look for and make use of structure.	Differentiate: $y = \frac{3x^2}{\sqrt{3x+1}}$ $y = 3 \sin^4(5x^2)$	Graphic Organizer: Provide students with colored paper, pens, and markers. Ask them to create a chart, a foldable card, or other creative method to organize all the derivative rules. For each rule, have them include the mathematical definition, examples, pictures, and helpful hints to understand and remember the rule.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use implicit differentiation to find the derivative of a function.	SMP 1 Make sense of problems and persevere in solving them.	Find the equation of the tangent and normal lines to $x^2 + y^2 = 36$ at $(5, \sqrt{11})$	Teacher-led discussion of need for implicit process and connection to chain rule; scaled examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Find a related rate and use related rates to solve real-life problems.</p>	<p>SMP 4 Model with mathematics.</p>	<p>The bottom of a 26 foot ladder is sliding along the ground at the speed of 3 feet per minute. Find the rate at which the ladder is sliding down the wall when it is 24 feet from the wall.</p>	<p>Round Table: Give students different related rates problems and a paper divided into five sections, titled as following:</p> <ul style="list-style-type: none"> ● Draw a picture ● Equation ● Derivative ● Specific information used ● Interpretation <p>Students first draw a picture of the situation and pass the papers clockwise. Students then critique the work in the previous section, complete the next section, and pass the papers again until all sections are completed.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● AP Classroom Assessment (Units 1 & 2) 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Derivatives Common Assessment <p>Performance Assessment:</p> <ul style="list-style-type: none"> ● Related Rates Task 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 3 Applications of Differentiation	Timeframe/Pacing: 21 Days
Essential Questions <ul style="list-style-type: none">• What do derivatives tell us about the behavior of values in a graph?• How can we piece together many different numerical characteristics to sketch a picture of a numerical relationship?• How can we use derivatives in a context to make improvements of systems?	
Enduring Understandings <ul style="list-style-type: none">• The first derivative indicates direction and the second derivative indicates shape• The combination of skills acquired through Math Analysis and Differential Calculus describe the trends, shape, direction and key features of a graph.• Optimization is a process making use of derivatives in order to help find maximal or minimal values to make systems better.	
Standards Taught and Assessed <ul style="list-style-type: none">• F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases• F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.• F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.• G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none">• ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.• ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.• Computer Science & Design Thinking:8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none">• 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment

- Determine for which value(s) of x $f'(x) > 0$, $f''(x) > 0$
- Given a picture of a graph, identify where the graph is increasing, decreasing, and any relative maxima, minima.
- Identify asymptotes of a function using limits.
- Write a function of a single variable to describe a geometric object with a constraint.

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Apply the Extreme Value Theorem to find extrema on a closed interval.	SMP 5 Use appropriate tools strategically.	Sketch a function on a closed interval which has a minimum at (2, 3) and a maximum at (-1, 4).	Create a Plan Provide students with a function represented analytically on a closed interval. Ask them to discuss and write x-values that are viable candidates for absolute extrema. Once they have established the viable candidates, ask them to design a method for analyzing	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			the behavior of the function's graph at the candidates and for identifying the extrema.	
Identify relative extrema on an open interval.	SMP 3 Construct viable arguments and critique the reasoning of others.	Sketch a function on a closed interval which has a relative minimum at (2, 3) but no relative maximum.	Class discussion and paired practice solving similar examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply Rolle's Theorem.	SMP 5 Use appropriate tools strategically.	Determine whether Rolle's Theorem can be applied to the function $f(x) = \tan x$ on the interval $[0, \pi]$.	Teacher-led discussion of Rolle's Theorem and class examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Mean Value Theorem.	SMP 5 Use appropriate tools strategically.	Find the point (s) guaranteed by the Mean Value Theorem on $[-\pi/2, \pi/2]$ for $f(x) = x - \cos x$.	Teacher-led discussion of the Mean Value Theorem and class examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine intervals on which a function is increasing or decreasing.	SMP 8 Look for and express regularity in repeated reasoning.	Identify all intervals on which the function $f(x) = -3x^2 - 4x - 2$ is	Critique Reasoning: Arrange students in groups of four to six, provide them with	Extended time, use of calculator, challenge work and specific other

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

		increasing, decreasing and constant.	a function's derivative (e.g., $g(x) = 5x + 3$), and ask them to determine if $g(x)$ is increasing or decreasing at a specific x -value, for example, $x = -3$. Ask students to share the reasoning for their conclusion with classmates in their group. Members of the group can then provide feedback and suggestions.	accommodations/modifications per a student's IEP or 504 plan.
Apply the First Derivative Test to find relative extrema of a function.	SMP 5 Use appropriate tools strategically.	Identify all relative extrema for the function $f(x) = -3x^2 - 4x - 2$ using the first derivative test.	Teacher-led discussion establishing criteria and conclusions of First Derivative Test; scaled examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine intervals on which a function is concave upward or downward.	SMP 5 Use appropriate tools strategically.	Identify all intervals on which the function $f(x) = -3x^2 - 4x - 2$ is concave up, down, and where there exists a point of inflection.	Connective problem using velocity and acceleration of a projectile to examine connection of second derivative; teacher-led examples illustrating a reliable process.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Second Derivative Test to find relative extrema of a	SMP 5 Use appropriate tools strategically.	Identify all relative extrema for the function $f(x) = -3x^2 - 4x - 2$	Think-Pair-Share: Provide students with a graph of f' and a graph of f'' . Ask	Extended time, use of calculator, challenge work and specific other

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

function.		using the second derivative test.	them to identify relative extrema and practice writing justifications for relative extrema using the first or second derivative test. Once they've written their justification, ask them to pair with a partner and share their justifications. Students can then discuss similarities or differences in their justification wording.	accommodations/modifications per a student's IEP or 504 plan.
Apply L'Hopital's Rule to evaluate a limit.	SMP2 Reason abstractly and quantitatively.	Determine the end behavior of the function $f(x) = \frac{x}{\sqrt{x^2 + 1}}$	Teacher-led discussion of definition and requirements for L'Hopital's Rule; student practice solving mixed problems.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Analyze and sketch the graph of a function.	SMP 5 Use appropriate tools strategically.	Identify intercepts, symmetry, domain, asymptotes, extrema, concavity and sketch a detailed graph of $f(x) = \frac{x}{\sqrt{x^2 + 2}}$	Predict and Confirm Provide students with the graph of a differentiable function, for example, $f(x) = x^3 - 4x^2 + 4x + 1$, but do not provide the rule for the function. Ask students to sketch a graph of the derivative of the function. Once students are done, reveal the rule for $f(x)$. Ask students	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

**Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum**

			to calculate $f'(x)$, and use technology to graph $f'(x)$ and compare it to their sketched graph.	
Solve applied minimum and maximum problems.	SMP 4 Model with mathematics.	A dairy farmer plans to fence in a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?	Recall activity modeling a geometric situation; Elicit connection to finding maxima and minima earlier in the unit to develop a process; class discussion and scaled examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the differential of a given function.	SMP 2 Reason abstractly and quantitatively.	For a given function, Find dy when $x = 1$ and $dx = 0.01$. Compare the values of dy and Δy .	Teacher-led discussion of tangent line approximations and instances where they would be useful. Detailed examples of all types; student practice	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Estimate propagated error using a differential.	SMP 1 Make sense of problems and persevere in solving them.	The measurement of the radius of the end of a log is found to be 14 inches, with a possible propagated error of $\frac{1}{4}$ inch. Use differentials to approximate the possible propagated error in computing the area of the end of the log.	Teacher-led discussion of examples of propagated error problems; connection with exact answers should be explored; paired problem solving.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Benchmark Assessment <ul style="list-style-type: none">• Not applicable	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none">• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Summative Assessment(s) <ul style="list-style-type: none">• Applications of Differentiation Common Assessment Performance Assessment: <ul style="list-style-type: none">• Curve Sketching Task	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 4 Integration	Timeframe/Pacing: 23 Days
Essential Questions <ul style="list-style-type: none">● How can we measure the space under a curve?● What can the area under a function tell us, in context?● How can we reverse derivative rules?	
Enduring Understandings <ul style="list-style-type: none">● A series of rectangles can be used to approximate the area under a curve.● The area under a rate function represents the amount of change of that quantity over time.● An antiderivative can be found by reversing the algorithm for the power rule and/or chain rule given a certain structure.	
Standards Taught and Assessed <ul style="list-style-type: none">● F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.● F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none">● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.● ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.● Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none">● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions	
Social Emotional Learning Competencies <ul style="list-style-type: none">● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Pre-Assessment <ul style="list-style-type: none"> Find a function whose derivative is $4x^{\frac{1}{4}}$ Using a series of 4 rectangles, approximate the area under $f(x) = x^2 + 1$ on $[0, 2]$. 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Write the general solution and particular solution of a differential equation.	SMP7 Look for and make use of structure.	Solve $\frac{dy}{dx} = 3x$ with initial condition $(1, 4)$.	Discussion with examples; Emphasis on connection between differentiation and solving a differential equation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use basic integration rules to find antiderivatives.	SMP 5 Use appropriate tools strategically.	$\int \frac{x+6}{\sqrt{x}} dx$	Collaborative chart of different forms of antiderivatives by inspection; Discussion with examples of generalized rules.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use sigma notation to write and evaluate a sum.	SMP7 Look for and make use of structure.	Express $1.2+1.5+1.8+\dots+4.2$ using sigma notation.	Warm-up activity to recall properties of sigma notation; class discussion	Extended time, use of calculator, challenge work and specific other accommodations/modific

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

				ations per a student's IEP or 504 plan.
Find the area of an enclosed region using the limit definition.	SMP 2 Reason abstractly and quantitatively.	Use the limit process to find the area under the curve $f(x) = x^2 + 1$ on $[0, 2]$.	Scaffolded activity to elicit different components of the limit process; teacher-led discussion of the entire process followed by student example.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the definition of a Riemann sum.	SMP2 Reason abstractly and quantitatively.	Write the following expression as a definite integral: $\lim_{n \rightarrow \infty} \sum_{i=1}^n 3 \left(\frac{i}{n} + 2 \right)^2 \left(\frac{1}{n} \right)$	Teacher-led discussion of definition of a definite integral	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate a definite integral using limits and geometric formulas.	SMP 5 Use appropriate tools strategically.	$\int_0^3 \sqrt{9 - x^2} dx$	Present the class with several examples of definite integrals set equal to Riemann sums in summation notation. Ask students to take five minutes to identify and write about all common elements between the two expressions and why they think the two expressions are equivalent. After finishing the five	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			minutes, ask students to share their observations with the class.	
Evaluate a definite integral using the properties of definite integrals.	SMP 5 Use appropriate tools strategically.	<p>Given $\int_0^3 f(x) dx = 2$,</p> <p>$\int_3^5 f(x) dx = -3$,</p> <p>find:</p> <p>$\int_0^5 2f(x) dx$</p>	Class discussion and paired practice solving similar examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate a definite integral using the Fundamental Theorem of Calculus.	SMP2 Reason abstractly and quantitatively.	$\int_0^5 (2x^2 + 3x + 1) dx$	Scaffolded activity to connect the limit process for finding area with area under a curve for a particle in motion with antiderivative of a velocity function to determine change in position.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Understand and use the Mean Value Theorem for Integrals.	SMP2 Reason abstractly and quantitatively.	Explain the Mean Value Theorem for Integrals graphically.	Visualizations using Geometer's Sketchpad & Calculus In Motion for MVT for Integrals and Average Value of a function.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Find the average value of a function over a closed interval.	SMP1 Make sense of problems and persevere in solving them.	Find the average value of the function $f(x) = x^2 + 1$ on $[-1, 2]$.	Visualizations using Geometer's Sketchpad & Calculus In Motion for MVT for Integrals and Average Value of a function.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Second Fundamental Theorem of Calculus.	SMP8 Look for and express regularity in repeated reasoning.	$\frac{d}{dx} \int_0^{x^2} (2t^2 + 3t + 1) dt$	Scaffolded example of integrating and then differentiating an accumulation function to elicit the general rule for FTCII.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Net Change Theorem.	SMP1 Make sense of problems and persevere in solving them.	Given $v(t) = \sin(2t) + 2$, Find the displacement and total distance traveled by a particle on $[1, 5]$.	Paired practice solving past AP FRQ's involving flow rates (ie: 2016 FRQ1)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use pattern recognition to find an indefinite integral.	SMP7 Look for and make use of structure.	$\int 6x\sqrt{3x^2 + 1} dx$	Look For a Pattern: Present students with several indefinite integrals and proposed, yet incorrect, Antiderivatives. Ask them to check the antiderivatives by	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			differentiating each and comparing to the original integrands. As students see that each antiderivative is incorrect, ask them to identify a pattern within the errors. Identifying this pattern will establish the foundation for integrating using substitution.	or 504 plan.
Use change of variables to find an indefinite integral and a definite integral	SMP7 Look for and make use of structure.	$\int_0^2 6x\sqrt{3x^2 + 1} dx$	Warm-up activity asking students to analyze structure of each antiderivative and theorize pattern to find antiderivatives. Scaled examples as class discussion.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate a definite integral involving an even or odd function.	SMP7 Look for and make use of structure.	$\int_{-2}^2 \sin x \cos x dx$	Graph even and odd functions to make a conclusion about properties of definite integrals.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Approximate a definite integral using the Trapezoidal Rule.	SMP 7 Look for and make use of structure.	Approximate the definite integral using the Trapezoidal Rule with $n=4$.	Class discussion and paired practice solving similar examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

		$\int_0^2 \sqrt{1+x^3} dx$		or 504 plan.
Benchmark Assessment <ul style="list-style-type: none"> • AP Classroom Assessment (Units 3 & 4) 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> • Integration Common Assessment Performance Assessment: <ul style="list-style-type: none"> • Net change task (AP FRQ) 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 5 Logarithmic, Exponential and Other Transcendental Functions	Timeframe/Pacing: 24 days
Essential Questions <ul style="list-style-type: none">● How can we find rates of change of transcendental functions?● How can we find the area under a transcendental function?● How do properties of inverse functions allow us to build connections and find otherwise unknown quantities?	
Enduring Understandings <ul style="list-style-type: none">● Properties of exponents, logarithms and inverse functions allow us to use implicit differentiation to establish new relationships and models.● The relative rate of change of two quantities is consistent with that of their derivatives.● Trigonometric connections can be used to solve integrals.	
Standards Taught and Assessed <ul style="list-style-type: none">● F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.● F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none">● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.● ELA: SL.11-12.5 Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.● Computer Science & Design Thinking: 8.1.12.DA.1 Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none">● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions	
Social Emotional Learning Competencies <ul style="list-style-type: none">● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Pre-Assessment <ul style="list-style-type: none"> ● Find the inverse of a function. ● Solve a half-life problem. ● Use the limit definition of the derivative to find the derivative of $y = e^x$. 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> ● Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Develop and use properties of the natural logarithmic function.	SMP2 Reason abstractly and quantitatively.	Use the properties of logarithms to expand $\ln z(z - 1)^2$	Warm-up activity to recall properties of logarithmic functions; class discussion.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find derivatives of functions involving the natural logarithmic function.	SMP7 Look for and make use of structure.	Find the derivative of $f(x) = \ln\left(\frac{x}{x^2 + 1}\right)$	Error Analysis: Assign a function to each student. Ask them to find the function's derivative using one or more derivative rules. Allow them to check their answers. Ask half of the class to redo their work to include an error, thus having the wrong answer. Ask students to record their correct or incorrect work on a card. Mix up	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

**Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum**

			the cards and redistribute, having students determine if the answer is correct or incorrect. If incorrect, they should explain what error was made, and find the correct answer.	
Use the Log Rule for Integration to integrate a rational function.	SMP7 Look for and make use of structure.	Find the indefinite integral: $\int \frac{x^2 + 2x + 3}{x^3 + 3x^2 + 9x} dx$	Class discussion and paired practice solving similar examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Integrate trigonometric functions.	SMP7 Look for and make use of structure.	Find the indefinite integral: $\int \csc 2x dx$	Class discussion and paired practice solving similar examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Verify that one function is the inverse of another function and determine whether a function has an inverse function.	SMP3 Construct viable arguments and critique the reasoning of others.	Show that f and g are inverses functions. a) analytically b) Graphically	Student recall activity; student responses on board; class discussion and critique of methods	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

		$f(x) = \sqrt{x-4}$ $g(x) = x^2 + 4, \quad x \geq 0$		
Find the derivative of an inverse function.	SMP8 Look for and express regularity in repeated reasoning.	Verify that f has an inverse. Then use the function f and the given real number a to find $(f^{-1})'(a)$. $f(x) = x^3 - 1, \quad a = 26$	Scaffolded exploration to elicit the relationship between rates of change on inverse functions. Generalize process through use of examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Differentiate natural exponential functions.	SMP7 Look for and make use of structure.	Find the derivative of: $y = x^2 e^{-x}$	Use of pre-existing knowledge of differentiation techniques and properties of logarithms/exponentials to derive method.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Integrate natural exponential functions.	SMP7 Look for and make use of structure.	Integrate: $\int \frac{e^{1/x^2}}{x^3} dx$	Use of pre-existing knowledge to have student derive a method to work backwards (u-substitution).	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Differentiate and integrate exponential functions that have bases other than e.</p>	<p>SMP7 Look for and make use of structure.</p>	<p>Integrate:</p> $\int \frac{3^{2x}}{1 + 3^{2x}} dx$	<p>Student activity: use logarithmic differentiation and implicit differentiation, in pairs, to derive formulas for derivatives for logarithms and exponentials of any base.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Use exponential functions to model compound interest and exponential growth.</p>	<p>SMP1 Make sense of problems and persevere in solving them.</p>	<p>A lake is stocked with 500 fish, and the population increases according to the logistic curve.</p> $p(t) = \frac{10,000}{1 + 19e^{-t/5}}$ <p>where t is measured in months.</p> <p>a) What is the limiting size of the fish population?</p> <p>b) At what rates is the fish population changing at the end of 1 month and at the end of 10 months?</p> <p>c) After how many months is the population increasing most rapidly?</p>	<p>Teacher-led discussion of population models and other growth/decay problems. Student participation to incorporate and connect previous knowledge to solve applications.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Apply L'Hopital's Rule to evaluate a limit.</p>	<p>SMP7 Look for and make use of structure.</p>	<p>Evaluate the limit:</p> $\lim_{x \rightarrow \infty} \frac{x^3}{e^{x^2}}$	<p>Teacher-led discussion to present criteria. Visualize the rule using Desmos or graphing utility to graphically verify its conclusion. Scaled examples.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Differentiate an inverse trigonometric function.</p>	<p>SMP7 Look for and make use of structure.</p>	$\frac{d}{dx} [\arctan(5 + 2x^2)]$	<p>Graphic Organizer: Provide students with colored paper, pens, and markers. Ask them to create a chart, a foldable card, or other creative method to organize all the derivative rules. For each rule, have them include the mathematical definition, examples, pictures, and helpful hints to understand and remember the rule.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Integrate functions whose antiderivatives involve inverse trigonometric functions.</p>	<p>SMP7 Look for and make use of structure.</p>	$\int \frac{-12}{1+9x^2} dx$	<p>Class discussion and paired practice solving similar examples.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Use the method of completing the square to integrate a function.</p>	<p>MP7 Look for and make use of structure.</p>	$\int \frac{dx}{x^2-2x+2}$	<p>Quiz-Quiz-Trade: Give students a card containing a question and have them write the answer on the back. Students then circulate around the room and find a partner. One student quizzes the other by showing only the side of the card with the question on it, and then they reverse roles. They swap cards, find a new partner, quiz each other, and the process continues.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> • Not applicable. 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> • Unit 5 Common Assessment <p>Performance Assessment:</p> <ul style="list-style-type: none"> • Determine rate of change of investment with a fixed interest rate. 				

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 6 Differential Equations	Timeframe/Pacing: 11 days
Essential Questions <ul style="list-style-type: none">● How can we visualize the connection between rate and amount?● How can we use information about rates to determine the amount of change?● How can rate relationships be used to solve real-world problems?	
Enduring Understandings <ul style="list-style-type: none">● A slope field contains information about patterns and connections between rate and amount.● Solving a differential equation results in a function that can be used to measure quantities.● Differential equations occur in nature and in the world and give us many of the formulas taught and used in earlier math courses.	
Standards Taught and Assessed <ul style="list-style-type: none">● F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.● F-BF.A.b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.● F-BF.A.c Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.● F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none">● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.● ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.● Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none">● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Social Emotional Learning Competencies <ul style="list-style-type: none"> 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness). 				
Pre-Assessment <ul style="list-style-type: none"> Find a function whose derivative is $\frac{dy}{dx} = 3y$. Sketch the graphs of a function and its derivative on the same plane and identify connections. 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Verify solutions to a differential equation and find particular solutions	SMP6 Attend to precision.	Determine whether the function $y = 3\cos 2x$ is a solution to the differential equation $y^4 - 16y = 0$	Teacher-led discussion of different methods followed by student practice examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Create and match slope fields for a given differential equation	SMP5 Use appropriate tools strategically.	Draw a slope field on a grid for $\frac{dy}{dx} = 3x + y$.	Create slope fields both by hand and using Desmos for more complicated examples to explore relationships between differential equations and path of function. <u>Collegeboard supplemental activities</u>	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Use separation of variables to solve a differential equation	SMP7 Look for and make use of structure.	Solve $y' = \frac{2x}{y}$.	Teacher demonstration of method; guided examples; paired practice	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve exponential growth and decay problems modeled by differential equations (including Newton's Law of Cooling)	SMP4 Model with mathematics.	If the rate of change of population is proportional to its size and the initial size is 100 insects which grows to 200 after 5 days, how many will there be at $t = 12$ days?	Teacher-led discussion of growth model; CollegeBoard supplemental problems	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
☉ Identify and solve homogeneous differential equations	SMP7 Look for and make use of structure.	Solve: $(x + y)dx - 2xdy = 0$	Teacher-led discussion of definitions, terminology and examples of process; Use of Desmos to visualize the equivalence of different forms of a valid solution.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<p>Benchmark Assessment</p> <ul style="list-style-type: none">● Benchmark Assessment on Units 5 & 6	<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none">● Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>Summative Assessment(s)</p> <ul style="list-style-type: none">● Differential Equations Common Assessment <p>Performance Assessment:</p> <ul style="list-style-type: none">● Population Growth Task	<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none">● Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 7 Applications of Integration	Timeframe/Pacing: 22 days
Essential Questions <ul style="list-style-type: none">• How can you use integrals to measure quantities and accumulations?• How can you use integrals to measure geometric objects?	
Enduring Understandings <ul style="list-style-type: none">• A definite integral, by definition, is the sum of its integrand over a closed interval.• Integrating length and area over a closed interval gives area and volume, respectively.	
Standards Taught and Assessed <ul style="list-style-type: none">• G-GMD.A. Explain volume formulas and use them to solve problems<ol style="list-style-type: none">1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.• B. Visualize relationships between two-dimensional and three-dimensional objects<ol style="list-style-type: none">4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.• F-IF.C. Analyze functions using different representations<ol style="list-style-type: none">7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.• N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none">• ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.• ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.• Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment

- Find the distance between two points in a plane: anywhere, horizontally, vertically
- Use a Riemann sum to approximate the area under a curve.
- Use a Riemann sum to approximate the volume of a solid.

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Find the area between two curves using integration.	SMP6 Attend to precision. SMP2 Reason abstractly and quantitatively.	Find the area of the region bounded by the graphs $f(x) = \frac{1}{9x^2}$, $y = 1$, $x = 1$, $x = 2$.	Recall definition of a definite integral and Reimann Sum. Elicit general formula for area between two curves.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Find the volume of a solid of revolution using the disk method.	SMP4 Model with mathematics	Find the volume of the solid generated by $y = \frac{1}{\sqrt{x}}$, between $x=1$ and $x=5$ rotated about the x -axis.	Use Geometer's Sketchpad Calculus in Motion Demos to visualize.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the volume of a solid of revolution using the washer method.	SMP4 Model with mathematics.	Find the volume of the solid generated by rotating the region bounded by $y = 6 - x^2$ and $y = x^2$ about the x -axis.	Use Geometer's Sketchpad Calculus in Motion Demos to visualize.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the volume of a solid with known cross sections.	SMP4 Model with mathematics.	The base of a solid of a solid is bounded by $y=x^3$, $y=0$ and $x=1$. Find the volume of the solid for each of the following cross sections (taken perpendicular to the y -axis): a)squares b)semicircles	In groups of four, each student has an identical paper with the same free-response question (e.g., 2015 AB #2(a)), along with four labeled boxes representing steps in the problem: <ul style="list-style-type: none"> ● Identify all points of intersection. ● Set up the integral(s). ● Integrate by hand. ● Integrate using a 	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

**Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum**

			<p>calculator.</p> <p>Have students complete the first step on their paper, and then pass the paper clockwise to another member in their group. That student checks the first step and then completes the second step on the paper. Students rotate again and the process continues until each student has their own paper back.</p>	
<p>Find the volume of a solid of revolution using the shell method.</p>	<p>SMP4 Model with mathematics.</p>	<p>Find the volume of the solid generated by rotating the region bounded by $y = 6 - x^2$ and $y = x^2$ about the y-axis.</p>	<p>Use Geometer's Sketchpad Calculus in Motion Demos to visualize.</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Determine when to use the shell method or washer method to find the volume of a solid of revolution..</p>	<p>SMP3 Construct viable arguments and critique the reasoning of others.</p>	<p>Find the volume of the solid generated by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the x-axis using both the Shell Method and Disc/Washer Method and explain which method is preferable.</p>	<p>Create cards with problems revolving around the x- or y-axis and others revolving around other axes (e.g., $y = x$ or $y = 3$). Give each student a card and have them write their answer on the back. Students quiz a partner about their own card then switch cards and repeat the process with a new partner. For the first round, concentrate on just setting</p>	<p>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			up the integrals (e.g., 2009 AB Form B #4(c), 2010 AB/BC #4(b), 2011 AB #3(c), and 2013 AB #5(b)). In the second round, students can use their calculators to find the volume (e.g., 2001 AB #1(c), 2006 AB/BC #1(b), 2007 AB/BC #1(b), and 2008 AB Form B #1(b)).	
Find the arc length of a smooth curve both with and without the use of technology.	SMP7 Look for and make use of structure.	Find the arc length of $y = \frac{x^7}{14} + \frac{1}{10x^5}$ on [1, 2]	Teacher-led proof of arc length using integration using distance formula and definition of definite integral.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment <ul style="list-style-type: none"> Not applicable. 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> Volume of Revolution Common Assessment Performance Assessment: <ul style="list-style-type: none"> Volume Task 				

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 8 Connections between Calculus Topics	Timeframe/Pacing: 26 days
Essential Questions <ul style="list-style-type: none">● How can limits be used to explore relationships between quantities?● How can we find rates of change and what can they be used to measure?● How can we find antiderivatives and what can they be used to measure?	
Enduring Understandings <ul style="list-style-type: none">● Limits can help us determine otherwise unknown quantities when real numbers fail to exist.● Derivatives (and their rules and properties) find rates of change and help optimize systems.● Integrals (and methods of integration) can be used to model systems and determine quantities of accumulation.	
Standards Taught and Assessed <ul style="list-style-type: none">● F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.● F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.● G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).● F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.● F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.● G-GMD.A. Explain volume formulas and use them to solve problems 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.● G-GMD.B.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none">● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.● ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.● Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better	

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

understand real world phenomena, including climate change.				
Highlighted Career Ready Practices and 21st Century Themes and Skill				
<ul style="list-style-type: none"> 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions 				
Social Emotional Learning Competencies				
<ul style="list-style-type: none"> 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness). 				
Pre-Assessment				
<ul style="list-style-type: none"> Not Applicable 				
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Use derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval.	SMP 1 Make sense of problems and persevere in solving them.	<u>2017 Exam</u> - Question #2	Emphasis on relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus. <u>Past AP Calculus Free-Response Questions</u>	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use discrete values and limit concepts, definitions, formulas, and theorems in calculus, solve problems of	SMP 5 Use appropriate tools strategically.	<u>2018 Exam</u> - Question 5	<u>Past AP Calculus Free-Response Questions</u>	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or

**Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum**

continuity, differentiation, integration.				504 plan.
Analyze the behaviors of functions by relating limits to differentiation, integration, and relating each of these concepts to the others.	SMP 2 Reason abstractly and quantitatively.	<u>2019 Exam</u> - Question 1	<u>Past AP Calculus Free-Response Questions</u>	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment <ul style="list-style-type: none"> Past AP Exam Multiple Choice Simulation 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> See Benchmark Performance Assessment: <ul style="list-style-type: none"> Graded AP Free Response Task 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

Unit Title: 9 Integration Techniques	Timeframe/Pacing: 9 days
Essential Questions <ul style="list-style-type: none"> ● How can we identify patterns to find antiderivatives? ● How can we quantify the accumulation of space over an infinite interval? 	
Enduring Understandings <ul style="list-style-type: none"> ● Reversing and reorganizing known derivative rules gives processes for finding antiderivatives. ● Using limits, together with integrals, we can measure infinite spaces. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● F-BF.A.1. Write a function that describes a relationship between two quantities. ● A-SSE.A.1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. ● ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest. ● Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions 	
Social Emotional Learning Competencies <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness). 	
Pre-Assessment <ul style="list-style-type: none"> ● $\int \sqrt{3x+1} dx$ 	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> ● Extended time, use of calculator, challenge work and specific other

Key: ■ Major Cluster □ Supporting Cluster ⊙ Additional Cluster

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

<ul style="list-style-type: none"> • $\int x\sqrt{3x^2 + 1} dx$ • $\int x\sqrt{3x + 1} dx$ 		accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Find an antiderivative using integration by parts.	SMP7 Look for and make use of structure.	$\int x^2 \ln x dx$	Teacher-led derivation of Integration By Parts formula using known product rule.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use a tabular method to perform integration by parts.	SMP5 Use appropriate tools strategically.	$\int x^3 \sin x dx$	Think-pair-share to theorize patterns in repeated use of integration by parts.	
Evaluate an improper integral that has either an infinite limit of integration or an infinite discontinuity.	SMP2 Reason abstractly and quantitatively.	$\int_0^{\infty} x^2 e^{-x} dx$	Review basic limits and L'Hopital's Rule for indeterminate forms; Illustrate connection to a convergent infinite series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or

Key: ■ Major Cluster □ Supporting Cluster ⊙ Additional Cluster

Hillsborough Township Public Schools
AP Calculus AB Mathematics Curriculum

			for distinct values.	504 plan.
Evaluate mixed integrals.	SMP1 Make sense of problems and persevere in solving them.	$\int_0^1 \frac{x}{x^2 - 4} dx$	Solve problems on a worksheet of mixed types of integrals.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment <ul style="list-style-type: none"> End of Course Benchmark Assessment (If applicable) 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> Unit 9 Common Assessment Performance Assessment: <ul style="list-style-type: none"> Accumulated Value of a Variable Continuous Income Stream Task 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: ■ Major Cluster □ Supporting Cluster ⊙ Additional Cluster

Bibliography

Supplemental Materials/Resources:

Larson, R., Edwards, B. H. (2018). *Calculus AP Edition*. Boston: Cengage Learning.

Digital Textbook and Materials:

www.webassign.com